Overview of Time Invariant Linear Feedback Control System Design Approaches

Will Kuhnle

Dallas Personal Robotics Group
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Overview of Time Invariant Linear Feedback Control System Design Approaches

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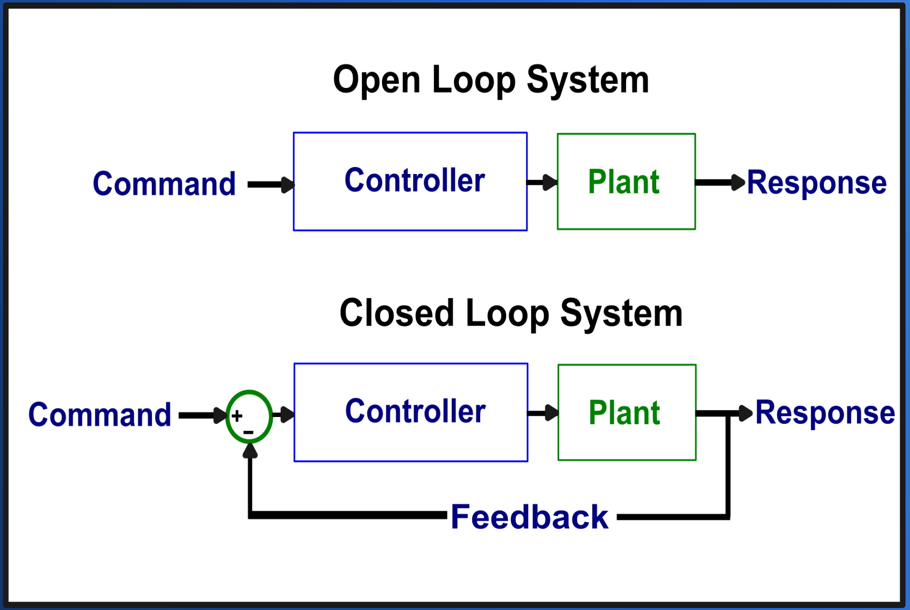
- OVERVIEW - "View from 30 thousand feet"
- plus a closer look at a few weeds.

TIME INVARIANT - Works the same today as it did yesterday.

- LINEAR -- If $y = f(x)$ and $x = a + b$
- then $f(a + b) = f(a) + f(b)$
- FEEDBACK CONTROL SYSTEM –
- A picture is worth many words --->
- DESIGN APPROACH
- 7 Simple Easy Steps ...



Overview of Time Invariant Linear Feedback Control System Design Approaches – Seven Steps

Design Approaches – Different Ways To Design System

Design procedure outline

1) Start study of problem to be solved by feedback system.
2) Translate problem objectives into performance specifications for closed loop system.
3) Develop math model for system PLANT, or "identify" PLANT.
4) Translate closed loop specifications into values for design parameters.
5) Design controller that will achieve design parameters.
6) Simulation to verify controller and plant performs per the closed loop specifications.
7) Build system and test it to verify that system solves problem.

1 Study The Problem

- What are you trying to accomplish?
- Why use closed loop control? Closed loop versus open loop
  - Reduces effect of variations in plant parameters.
  - Reduces effect of disturbances.
  - Increases speed of response.
  - Stabilize unstable open loop system.
- How fast does system need to respond?
- What steady state accuracy is required?
- Can you describe the disturbances?
- YOU CANNOT CONTROL WHAT CANNOT BE MEASURED !!!
- Looking at closed loop equations vs open loop ...

1 Study The Problem – Transfer Function

U = Input

D = Disturbance,

G = Plant Transfer Function

Y = Output

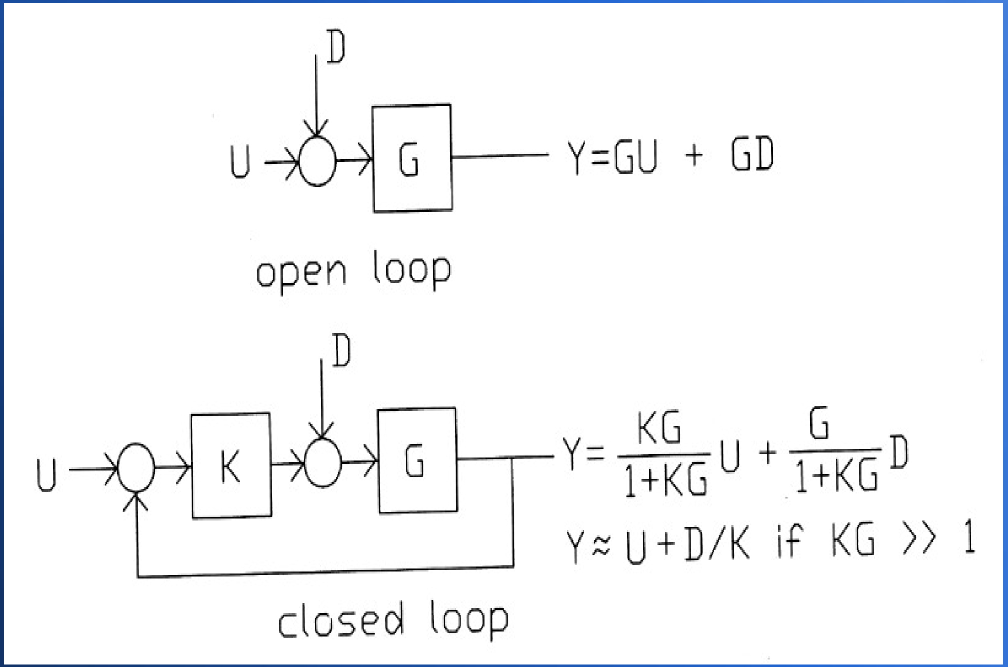
Closed Loop Output Derivation (Solving for Y)

Y = [(U-Y)K) + D]G = UKG – YKG + DG

Y + YKG = UKG + DG

(1+KG) Y = UKG + DG

Y = [KG / (1+ KG)]U + [G/(1+KG)]D



2 Performance Specifications For Closed Loop System

Transient response to a step input.

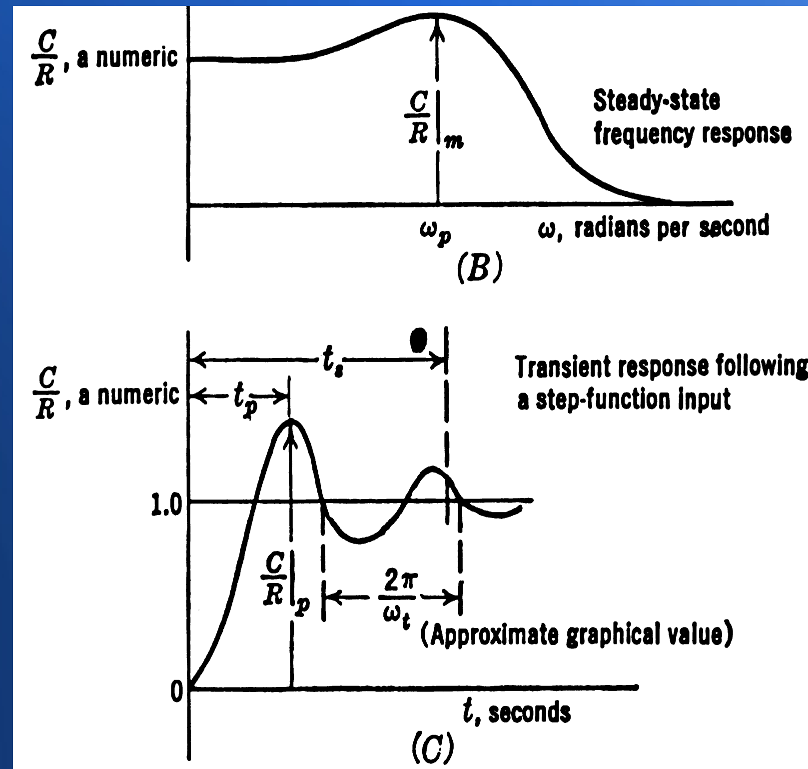
- Rise time
  - Time to go from 10% to 90% of steady state value.
  - Time to go to 63% of steady state value.
  - Time it takes to get to steady state value 1st time
- Overshoot
  - Difference between peak value and steady state value.
- ts
  - Minimum time for output to stay within 5% of steady state value.
- tp
  - Time to go to peak value.

Steady state frequency response

- Bandwidth
  - Frequencey at which the gain is down 3db from DC gain.
- Closed loop damping factor
  - Measure of maximum gain to DC gain
  - Related to C/Rm in Figure.

Steady state error to different types of input.

- Type 0: u(t) = k = constant requires constant error signal.
- Type 1: u(t) = kt requires constant error signal.
- Type 2: u(t) = kt^2 requires constant error signal.



3 Develop Math Model for System Plant or Identify Plant

Design methods: Input-output vs State-space

Time: continuous vs discrete

Signal domain: time vs frequency

- Obtain model: experiment on plant vs first principles of physics

  Input-output-- "black box" approach

  single high order differential equation -- time domain

  Laplace transform to frequency domain

  G(s) = [(1+sT1)(1+sT2) / [s(1+T3)(1+sT4)]

- State-space -- plant is considered a state machine
- multiple 1st order differential equations -- time domain

  dx/dt = Ax + Bu y = Cx + D

3 Develop Math Model for System Plant – State Space DC Motor Example

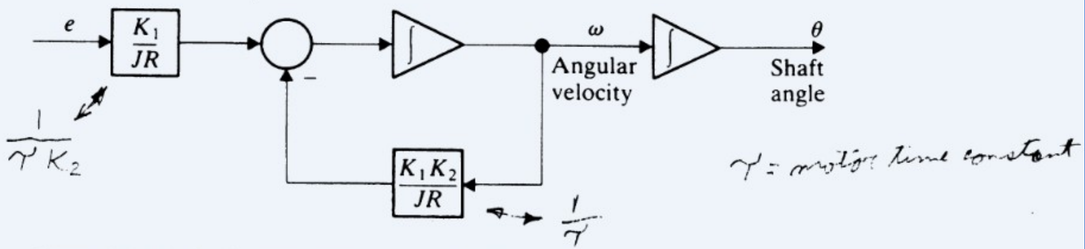


Figure 2.3 Block diagram representing dynamics of dc motor driving inertia load.

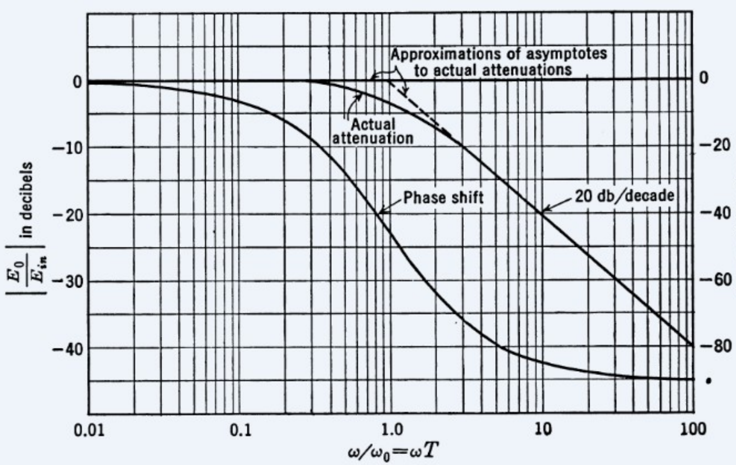
Equations (2B.7) and (2B.8) can be arranged in the vector-matrix form

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -K_1K_2/JR\end{bmatrix}\begin{bmatrix}\theta \\ \omega\end{bmatrix} + \begin{bmatrix}0 \\ K_1/JR\end{bmatrix}e$$

A block-diagram representation of the differential equations that represent this system is given in Fig. 2.3.

Friedland, B,
CONTROL SYSTEM DESIGN An Introductio to State-Space Methods,
Dover Publications, Inc., 2005

3 Develop Math Model for System Plant – Input Output DC Motor Example



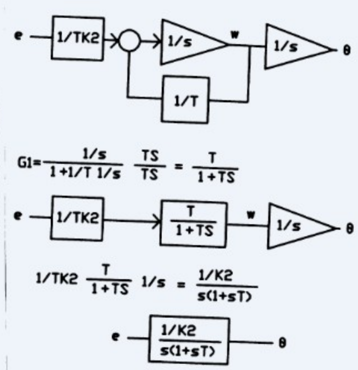


FIGURE 11.3–1. Plot of attenuation and phase shift of

$$\frac{E_o}{E_{in}} = \frac{1}{1 + j\omega T} = \frac{1}{1 + \frac{j\omega}{\omega_0}}$$

$$T = 1$$

showing approximation of asymptotes to actual attenuation.

Chestnut, H, and Mayer, R
SERVOMECHANISMS and REGULATING SYSTEM DESIGN, V1,
John Wiley & sons, Inc., 1951

4 & 5 Translate Closed Loop Specifications into Values for Design Parameters and Design Controller That Will Achieve Design Parameters - Input Output Design Method

- Input Output Design Method

Bode plot method: Required design parameters are shape of the OPEN LOOP frequency response curve of the plant and controller (Bode plot) that will have the required response WHEN THE LOOP IS CLOSED. Also, the gain-margin and the phase-margin. See Chestnut & Mayer book.

Controller design consists of selecting LEAD, LAG, and LEAD-LAG networks.

Pole placement method: Required parameters are the pole locations in the complex s or z plane. For a stable system continuous time s poles must be to the left (negative) of the imaginary axis. Discrete time z poles must be inside the unit circle centered on the origin.

Poles are values of s that zeroes the TRANSFER FUNCTION denominator.

$G(s) = K(1+sT2) / [s(1+sT1)(1+sT3)]$

$s = -1/T1$, & $s = -1/T3$ are poles. $s = -1/T2$ is a zero

4 & 5 Translate Closed Loop Specifications into Values for Design Parameters and Design Controller That Will Achieve Design Parameters - State Space Method

- State-Space Design Method

Pole placement: Required design parameters are essentially the same as for Input-output design.

The output from the controller is $u = Gx$. G is the gain matrix, x system state vector. Given the required pole locations calculating the G matrix is relatively straight forward.

Linear, Quadratic optimum control method: Required design parameters are the Q and R matrices.

Q is state weighting, R is control weighting.

"cost function" $$V = \int_t^T [x'(m)Qx(m) + u'(m)Ru(m)]\, dm$$

There is a procedure for calculating the gain matrix G that minimizes V. If any of the state variables are not measured on the physical plant (which is the usual case) then they are estimated by an OBSERVER.

An observer is a math model of the plant that is forced, by feedback from the plant output y, to track the plant output.

$$d\hat{x}/dt = A\hat{x} + B\hat{u} + K(y - C\hat{x})$$ An "optimum" observer is a Kalman filter.

Plant with Observer Generating Feedback

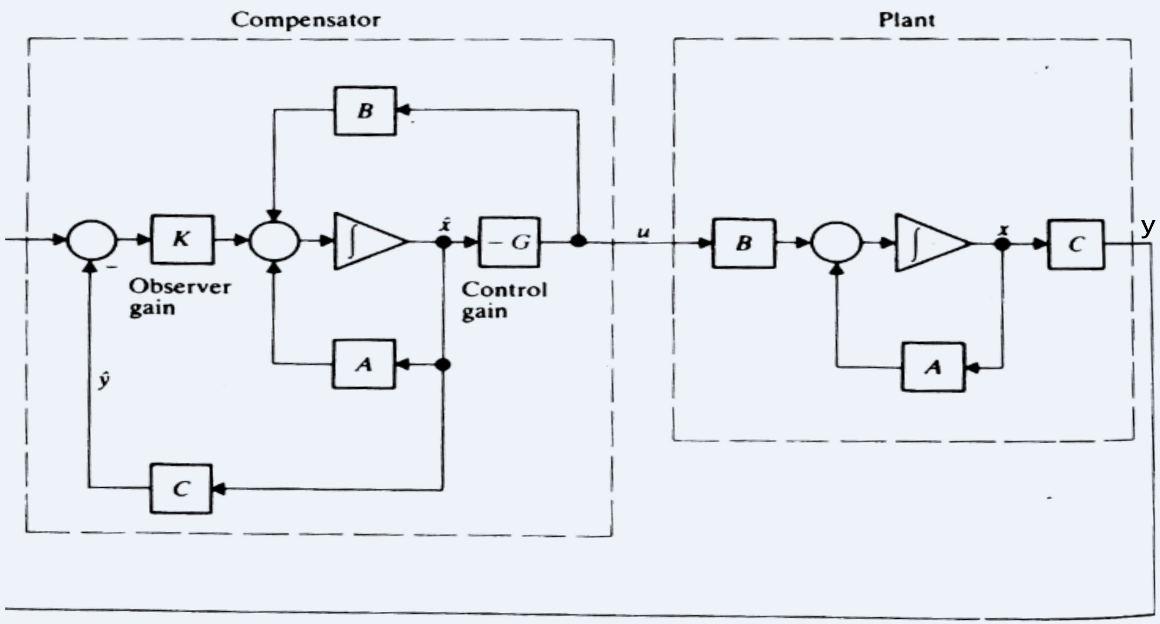


Figure 8.1 Control system using observer in compensator.

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